Convex Polytopes

All polytopes (polyhedra) here are assumed to be *convex*.

LA6⋄1. Prove that every face of a polytope P is contained in a facet (of codim 1).

LA6⋄2. Determine the faces of the n-simplex.

LA6⋄3. Given a 3-dimensional compact polytope such that every two vertices are adjacent, show that it is a tetrahedron.

LA6⋄4. Describe (in coordinates) the faces of the intersection of the n-dimensional cube $P = \{0 \leq x_k \leq 1 \mid k = 1, \ldots, n\}$ with the hyperplane $x_1 + \ldots + x_n = \frac{n}{2}$.

LA6⋄5. Prove that the convex hull of any finite set of points that are in *general position* in $\mathbb{R}^d$ (there are no $d+1$ points in one hyperplane) is a *simplicial* compact polytope, i.e. all of whose proper faces are simplices.

LA6⋄6. Show that if a compact polytope is both *simple* (a polytope in $\mathbb{R}^d$ is simple if every vertex belongs to exactly d facets) and simplicial, then it is a simplex or an n-gon.

LA6⋄7. Show that every compact polytope is affinely isomorphic to a bounded intersection of an orthant with an affine subspace.