Theorem 1) If M is a compact convex body, then $M = \operatorname{conv}\{\operatorname{ext}(M)\}$.

2) Let P be a convex polyhedron. Then
P is compact $\iff P = \operatorname{conv}\{v_1, \dots, v_n\} = \operatorname{conv}(V)$
(Moreover, $\operatorname{ext}(P) \subset V$.)

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Can remove from V such v_j that $v_j \in \operatorname{conv}(V \setminus v_j)$.

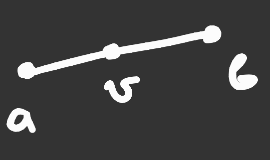
So if P is a compact polytope then
$P = \operatorname{conv}\{v_1, \dots, v_n\}$ where all $v_j \in \operatorname{ext}(P)$.

$$P = \bigcap_{j=1}^{N} H^-_{e_j}$$

Def A face of $P = F = P \cap H_{e_{j_1}} \cap \dots \cap H_{e_{j_s}}$.

Vertex = 0-dim face
Edge = 1-dim face
facet = codim-1 face (max proper face).

Theorem a) $\underbrace{\text{Vertices}}_{\text{of comp. polytopes}}$ are extreme points
b) $\operatorname{vert} F \subseteq \operatorname{vert} P$ c) if $\underset{\text{face}}{F_1} \subset \underset{\text{face}}{F_2} \subset P \Rightarrow F_1$ is a face of P.

Proof: 1) 

if a vertex $v \notin \operatorname{ext}(P)$ then $\exists a, b: \ v \in \operatorname{rel\,int}[a,b] = (a,b)$

But v is a face, so $\exists H_e$ that cuts $[a,b]$
(otherwise all H_e's contain $\langle [a,b] \rangle$ and then v is an interior point of a 1-dim face)

2) So $\operatorname{vert} P \subseteq \operatorname{ext} P \subset \partial P$. Also $\operatorname{ext} F \subset \operatorname{ext} P$
and $\operatorname{ext} F = \operatorname{ext} P \cap F$

3) $\forall x \in \partial P \ \exists$ face $F_{min} \ni x$ $\left(F_{min} = \bigcap \text{supporting hyperplanes for } x\right)$

4) if $v \in \text{ext } P$ and $\dim F_{min}(v) \geq 1$, then $v \in \text{rel int } F_{min}$ and thus $v \notin \text{ext } F_{min}$