

Euclidean and Hermitian Spaces

LA5◊1. There are three such vectors in the Euclidean space \mathbb{E}^3 that all their pairwise inner products are non-negative. Does there always exist such orthonormal basis in \mathbb{R}^3 that all these three vectors lie in one coordinate octant?

LA5◊2. Two vectors in the Euclidean vector space lie on the one side of the given hyperplane. The angle between these vectors is obtuse. Is it true that the angle between their orthogonal projections on this hyperplane is also obtuse?

LA5◊3. Find the maximum number of vectors that can be released from a given point in the Euclidean space \mathbb{R}^n so that all the pairwise angles between them are obtuse?

LA5◊4. Suppose $I^n = \{x \in \mathbb{R}^n \mid x_j \leq 1 \forall j\}$ is a standard n -dimensional cube. How much

- (a) k -dimensional faces
- (b) inner diagonals (between vertices which are symmetric to each other with respect to the center of cube)
- (c) hyperplanes of symmetries

it has?

LA5◊5. Is it possible that the intersection of the positive orthant

$$\{(x_1, x_2, x_3, x_4) \mid x_1, x_2, x_3, x_4 \geq 0\} \subset \mathbb{R}^4$$

with some 2-dimensional plane is a square?

LA5◊6. For which $c \in \mathbb{R}$ the hyperplane $\sum x_j = c$ intersects with

- (a) the three dimensional cube $I^3 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 \mid |x_j| \leq 1 \forall j\}$?
- (b) the four dimensional cube $I^4 = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid |x_j| \leq 1 \forall j\}$?

Draw all the polygons and polyhedra respectively that are cut from the cube by such hyperplanes.