Affine Spaces

LA2⋄1. Write the equation (in coordinates x_1, x_2) of a line in $\mathbb{A}^2$:

(a) passing through the point $(2, -3)$ and parallel to the vector $(5, 2)$;

(b) passing through the points $(-3, 5)$ and $(4, -1)$.

LA2⋄2. Suppose $P \neq Q \in \mathbb{A}^2$. Is it true that

$$f : X \mapsto \text{center}(P, Q, X)$$

is an affine map? Is it bijective?

LA2⋄3. Suppose an affine transformation $f\colon \mathbb{A}^2 \to \mathbb{A}^2$ maps each line to a line parallel to it or to the same line. Prove that f is either a parallel translation or a homothety.

LA2⋄4. Write the standard coordinate form of an affine transformation in $\mathbb{A}^2(\mathbb{R})$ that maps the point $(1, -2)$ to the point $(0, 10)$, and the lines $10x_1 - 4x_2 = 1$ and $3x_1 - 3x_2 = -7$ to the lines $x_1 - 2x_2 = -3$ and $x_1 - x_2 = 6$, respectively.

LA2⋄5. Suppose ℓ_1 and ℓ_2 are skew lines in the space $\mathbb{E}^3$. Is it true that lines PQ, where $P \in \ell_1$, $Q \in \ell_2$, sweep the whole space?

LA2⋄6. How many lines are there in $\mathbb{A}^2(\mathbb{F}_q)$ over the finite field $\mathbb{F}_q$ of q elements?

LA2⋄7. Describe an affine transformation $f \circ H_O^\lambda \circ f^{-1}$, where H_O^λ denotes a homothety with the center $O \in \mathbb{A}^2$ and the coefficient $\lambda \in \mathbb{R}$, and $f\colon \mathbb{A}^2 \to \mathbb{A}^2$ is some arbitrary affine transformation.

LA2⋄8. What is the composition $H_P^\lambda \circ H_Q^\mu\colon \mathbb{A}^2 \to \mathbb{A}^2$ of two homotheries with different centers and coefficients?

LA2⋄9. Let V be an affine space of dimension n over the finite field $\mathbb{F}_q$ of q elements. How many k-dimensional affine subspaces are there in V?