# Linear Algebra

## Lecture 3: Exercises on Affine Geometry and Bilinear Forms

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## Problem: Plane

$P \subset \mathbb{A}$ is a plane, iff for any $A, B \in P$ the line $AB$ also lies in $P$.

# Problem: Convex Combinations

For any $A_0, A_1, \dots, A_k \in M$, where $M$ is convex, $M$ also contains every convex combination $\sum \lambda_j A_j$.

# Convex Hull

For any $M \subset \mathbb{A}$, the set $\operatorname{conv}(M)$ of all convex combinations of points in $M$ is convex.

# Linear Functions on Polynomials

Prove that functions $\varphi_0, \varphi_1, \dots, \varphi_n$ defined as $\varphi_k(p) = p(x_k)$, form a basis in $\Bbbk^*[x]_n$, where $x_0, x_1, \dots, x_n \in \Bbbk$.

## Problem: Orthogonal Group

A subgroup of $\mathrm{GL}(n, \mathbb{R})$ that preserves the standard inner product is $\mathrm{O}(n, \mathbb{R})$:

$$\mathrm{O}(n, \mathbb{R}) = \{A \in \mathrm{GL}(n, \mathbb{R}) \mid (Ax, Ay) = (x, y)\}$$

# Problem: General Affine Group

$$\mathrm{Aff}(\mathbb{A}) = T(\mathbb{A}) \rtimes \mathrm{GL}(V)$$

## Problem: $\mathrm{Isom}(\mathbb{E}^n)$

$$\mathrm{Isom}(\mathbb{E}^n) = T(\mathbb{E}^n) \rtimes \mathrm{O}(n, \mathbb{R})$$

## Problem: $\mathrm{Isom}(\mathbb{E}^n)$ and Reflections

$\mathrm{Isom}(\mathbb{E}^n)$ is generated by reflections.

# Orthogonal Complement

If $\alpha$ is non-degenerate, then

$$\dim U^{\perp} = \dim V - \dim U \textit{ and } (U^{\perp})^{\perp} = U.$$