Linear Operators - 2

LA6⋄1. Find the matrix of linear operator P of orthogonal projection $P\colon \mathbb{R}^4 \to U$, where $U = \{x \in \mathbb{R}^4 \mid x_1 + x_2 + x_3 + x_4 = 0\}$.

LA6⋄2. Is it true that every matrix is conjugate to its transpose matrix?

LA6⋄3. Give an example of a nonidentical operator without a cyclic[1] vector.

LA6⋄4. Find the degree of the minimal polynomial of a square matrix of rank 1.

LA6⋄5. Suppose that the characteristic polynomial of a linear operator $F\colon V \to V$ is irreducible and has a degree d. Show that $\dim V = d$ and for every $v \in V \setminus \{0\}$ vectors $\{v, Fv, F^2v, \dots, F^{d-1}v\}$ make up the basis of V.

LA6⋄6. Is it true that operator $F\colon \mathbb{R}^n \to \mathbb{R}^n$ is nilpotent iff $\operatorname{tr} F^k = 0$ for all $1 \leq k \leq n$?

LA6⋄7. Let the degree of the minimal polynomial of a linear operator $F\colon V \to V$ is equal to $\dim V$. Is it true that every operator permutable with F is a polynomial of F?

LA6⋄8. Linear operator $F\colon \mathbb{R}^n \to \mathbb{R}^n$ has a matrix with numbers $\lambda_1, \dots, \lambda_n$ on the secondary diagonal and zeros in other places. When it can be diagonalized over $\mathbb{R}$?

LA6⋄9. Prove that for any positive definite symmetric linear operator A there is a unique positive definite symmetric linear operator B such that $A = B^2$.

LA6⋄10. (*Polar Decomposition*) Prove that each invertible operator A in a Euclidean space can be decomposed in so called 'polar decomposition'

$$A = S_1 O_1 = O_2 S_2,$$

where operators S_j are unique positive definite symmetric operators and O_j are unique orthogonal operators.

[1] A vector $v \in V$ is called cyclic for an operator A if $V = \langle A^m v \mid m \geq 0 \rangle$