Vector Spaces

LA1⋄1. Find a basis of the vector space $V = \{p(x) \in \mathbb{R}[x]_4 \mid p'(5) = 0\}$.

LA1⋄2. Find a dimension and a basis of the vector space
(a) $\mathrm{Mat}_n^+(\mathbb{R})$ of all symmetric matrices $A = A^T \in \mathrm{Mat}_n(\mathbb{R})$.
(b) $\mathrm{Mat}_n^-(\mathbb{R})$ of all skew-symmetric matrices $A = -A^T \in \mathrm{Mat}_n(\mathbb{R})$.
(c) $\mathfrak{sl}_n(\mathbb{R}) = \{A \in \mathrm{Mat}_n(\mathbb{R}) \mid \mathrm{tr}\, A := a_{11} + a_{22} + \ldots + a_{nn} = 0\}$.

LA1⋄3. Prove that $\mathrm{Mat}_n(\mathbb{R}) = \mathrm{Mat}_n^+(\mathbb{R}) \oplus \mathrm{Mat}_n^-(\mathbb{R})$

LA1⋄4. Give an example of a finite dimensional space V and three its pairwise transversal subspaces U, W, T (that is, intersecting only at the origin) such that

$$\dim U + \dim W + \dim T = \dim V,$$

but $U + W + T \neq V$.

LA1⋄5. Is it true that

$$\dim(U+V+W) = \dim U+\dim V+\dim W-\dim(U\cap V)-\dim(U\cap W)-\dim(W\cap V)+\dim(U\cap V\cap W)?$$

LA1⋄6. Suppose $\dim(U + V) = \dim(U \cap V) + 1$ for some two vector subspaces $U, V \subset \mathbb{R}^n$. Is it true that $U + V$ is equal to one of the subspaces U, V and $U \cap V$ is equal to another?

LA1⋄7. Let V be a vector space of dimension n over the finite field $\mathbb{F}_q$ of q elements. How many
(a) vectors (b) bases (c) k-dimensional subspaces
are there in V?

LA1⋄8. Prove that the vector space of all continuous functions on $\mathbb{R}$ is infinite dimensional.